NCERT Solutions Class 10 Maths Chapter 5: Arithmetic Equations

Ex 5.1 Question 1.

In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?
(i) The taxi fare after each km when the fare is $\mathrm{Rs} 15$ for the first km and $\mathrm{Rs} 8$ for each additional $\mathbf{km}$.
(ii) The amount of air present in a cylinder when a vacuum pump removes one fourth of the air remaining in the cylinder at a time.
(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.
(iv) The amount of money in the account every year, when Rs 10,000 is deposited at compound Interest at 8% per annum.

Answer.

(i)Taxi fare for $1 \mathrm{st} \mathrm{km}=\mathrm{Rs} 15$, Taxi fare after $2 \mathrm{~km}=15+8=\mathrm{Rs} 23$
Taxi fare after $3 \mathrm{~km}=23+8=\mathrm{Rs} 31$
Taxi fare after $4 \mathrm{~km}=31+8=$ Rs 39
Therefore, the sequence is $15,23,31,39 \ldots$
It is an arithmetic progression because difference between any two consecutive terms is equal which is 8 .
$(23-15=8,31-23=8,39-31=8, \ldots)$
(ii)Let amount of air initially present in a cylinder $=V$

Amount of air left after pumping out air by vacuum pump $=V-\frac{V}{4}=\frac{4 V-V}{4}=\frac{3 V}{4}$
Amount of air left when vacuum pump again pumps out air

$=\frac{3}{4} V-\left(\frac{1}{4} \times \frac{3}{4} V\right)=\frac{3}{4} V-\frac{3}{16} V=\frac{12 V-3 V}{16}=\frac{9}{16} V$

So, the sequence we get is like $V, \frac{3}{4} V, \frac{9}{16} V \ldots$
Checking for difference between consecutive terms ...
$\frac{3}{4} V-V=-\frac{V}{4}, \frac{9}{16} V-\frac{3}{4} V=\frac{9 V-12 V}{16}=\frac{-3 V}{16}$

Difference between consecutive terms is not equal.
Therefore, it is not an arithmetic progression.
(iii) Cost of digging 1 meter of well $=$ Rs 150

Cost of digging 2 meters of well $=150+50=$ Rs 200
Cost of digging 3 meters of well $=200+50=$ Rs 250
Therefore, we get a sequence of the form $150,200,250 \ldots$
It is an arithmetic progression because difference between any two consecutive terms is equal. $(200-150=250-200=50 \ldots)$

Here, difference between any two consecutive terms which is also called common difference is equal to 50 .
(iv)Amount in bank after Ist year $=10000\left(1+\frac{8}{100}\right)$ ..

Amount in bank after two years $=10000\left(1+\frac{8}{100}\right)^{2} \ldots$

$\text{Amount in bank after three years } =10000\left(1+\frac{8}{100}\right)^{3} \ldots$

Amount in bank after four years $= 10000\left(1 + \frac{8}{100}\right)^4 \ldots$
It is not an arithmetic progression because (2) - (1) $\neq$ (3) - (2)
(Difference between consecutive terms is not equal)
Therefore, it is not an Arithmetic Progression.

Ex 5.1 Question 2.

Write first four terms of the AP, when the first term a and common difference d are given as follows:
(i) $a = 10, d = 10$
(ii) $\mathbf{a} = -2,\ \mathrm{d} = 0$
(iii) $\mathbf{a} = 4,\ \mathrm{d} = -3$
(iv) $a = -1, d = \frac{1}{2}$
(v) $\mathrm{a} = -1.25,\ \mathrm{d} = -0.25$

Answer.

(i) First term $= a = 10, d = 10$
Second term $= a + d = 10 + 10 = 20$
Third term $=$ second term $+d = 20 + 10 = 30$
Fourth term $=$ third term $+d = 30 + 10 = 40$
Therefore, first four terms are: 10, 20, 30, 40
(ii) First term $= \mathrm{a} = -2,\ \mathrm{d} = 0$

Second term $= a + d = -2 + 0 = -2$
Third term $=$ second term $+d = -2 + 0 = -2$
Fourth term $=$ third term $+d = -2 + 0 = -2$

Therefore, first four terms are: $-2, -2, -2, -2$
(iii) First term $= a = 4, d = -3$

Second term $= a + d = 4 - 3 = 1$
Third term $=$ second term $+\mathrm{d} = 1 - 3 = -2$
Fourth term $=$ third term $+d = -2 - 3 = -5$

Therefore, first four terms are: $4, 1, -2, -5$
(iv) First term $= a = -1, d = \frac{1}{2}$

Second term $= \mathrm{a} + \mathrm{d} = -1 + \frac{1}{2} = -\frac{1}{2}$
Third term $=$ second term $+d = -\frac{1}{2} + \frac{1}{2} = 0$
Fourth term $=$ third term $+d = 0 + \frac{1}{2} = \frac{1}{2}$
Therefore, first four terms are: $-1, -\frac{1}{2}, 0, \frac{1}{2}$
(v) First term $= \mathrm{a} = -1.25,\ \mathrm{d} = -0.25$

Second term $= \mathrm{a} + \mathrm{d} = -1.25 - 0.25 = -1.50$
Third term $=$ second term $+\mathrm{d} = -1.50 - 0.25 = -1.75$
Fourth term $=$ third term $+\mathrm{d}$
$= -1.75 - 0.25 = -2.00$

Therefore, first four terms are: $-1.25, -1.50, -1.75, -2.00$
Ex 5.1 Question 3.

For the following APs, write the first term and the common difference.

(i) $3, 1, -1, -3 \ldots$
(ii) $-5, -1, 3, 7 \ldots$
(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$
(iv) $0.6, 1.7, 2.8, 3.9 \ldots$

Answer.

(i) $3, 1, -1, -3 \ldots$
First term $= a = 3,$
Common difference $(\mathrm{d}) =$ Second term - first term $=$ Third term - second term and so on

Therefore, Common difference $(d) = 1 - 3 = -2$
(ii) $-5, -1, 3, 7 \ldots$

First term $= a = -5$

Common difference $(d) =$ Second term - First term
$=$ Third term - Second term and so on
Therefore, Common difference $(d) = -1 - (-5) = -1 + 5 = 4$
(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

First term $= \mathrm{a} = \frac{1}{3}$
Common difference $(\mathrm{d}) =$ Second term - First term
$=$ Third term - Second term and so on

Therefore, Common difference $(d) = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$
(iv) $0.6, 1.7, 2.8, 3.9 \ldots$

First term $= \mathrm{a} = 0.6$
Common difference $(\mathrm{d}) =$ Second term - First term
$=$ Third term - Second term and so on
Therefore, Common difference $(d) = 1.7 - 0.6 = 1.1$

Ex 5.1 Question 4.

Which of the following are APs? If they form an AP, find the common difference d and write three more terms.
(i) $2, 4, 8, 16 \ldots$
(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \cdots$
(iii) $-1.2, -3.2, -5.2, -7.2 \ldots$
(iv) $-10, -6, -2, 2 \ldots$
(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots$
(vi) $0.2, 0.22, 0.222, 0.2222 \ldots$
(vii) $0, -4, -8, -12 \ldots$
(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \cdots$
(ix) $1, 3, 9, 27 \ldots$
(x) $a, 2a, 3a, 4a \ldots$
(xi) $a, a^2, a^3, a^4 \ldots$
(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$
(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$
(xv) $1^2, 5^2, 7^2, 73 \ldots$

Answer.

(i) $2, 4, 8, 16 \ldots$
It is not an AP because difference between consecutive terms is not equal.
As $4 - 2 \neq 8 - 4$
(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \cdots$

It is an AP because difference between consecutive terms is equal.
$\Rightarrow \frac{5}{2} - 2 = 3 - \frac{5}{2} = \frac{1}{2}$

Common difference $(\mathrm{d}) = \frac{1}{2}$
Fifth term $= \frac{7}{2} + \frac{1}{2} = 4$ Sixth term $= 4 + 1/2 = \frac{9}{2}$
Seventh term $= \frac{9}{2} + \frac{1}{2} = 5$
Therefore, next three terms are $4, \frac{9}{2}$ and 5 .
(iii) $-1.2, -3.2, -5.2, -7.2 \ldots$

It is an AP because difference between consecutive terms is equal.
$\Rightarrow -3.2 - (-1.2)$

$= -5.2 - (-3.2)$
$= -7.2 - (-5.2) = -2$

Common difference $(\mathrm{d}) = -2$
Fifth term $= -7.2 - 2 = -9.2$ Sixth term $= -9.2 - 2 = -11.2$
Seventh term $= -11.2 - 2 = -13.2$

Therefore, next three terms are $-9.2, -11.2$ and -13.2
(iv) $-10, -6, -2, 2 \ldots$

It isan AP because difference between consecutive terms is equal.
$\Rightarrow -6 - (-10) = -2 - (-6)$
$= 2 - (-2) = 4$

Common difference $(\mathrm{d}) = 4$
Fifth term $= 2 + 4 = 6$ Sixth term $= 6 + 4 = 10$
Seventh term $= 10 + 4 = 14$

Therefore, next three terms are 6, 10 and 14
(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 3+\sqrt{2}-3$

$=\sqrt{2}, 3+2\sqrt{2}-(3+\sqrt{2})$

$=3+2\sqrt{2}-3-\sqrt{2}=\sqrt{2}$

Common difference $(\mathrm{d})=\sqrt{2}$

Fifth term $=3+3\sqrt{2}+\sqrt{2}=3+4\sqrt{2}$

$\text{Sixth term } =3+4\sqrt{2}+\sqrt{2}=3+5\sqrt{2}$

$\text{Seventh term } =3+5\sqrt{2}+\sqrt{2}=3+6\sqrt{2}$

Therefore, next three terms are $(3+4\sqrt{2})(3+5\sqrt{2})=(3+6\sqrt{2})$

(vi) $0.2, 0.22, 0.222, 0.2222\ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 0.22-0.2 \neq 0.222-0.22$

(vii) $0,-4,-8,-12\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow -4-0=-8-(-4)$

$=-12-(-8)=-4$

Common difference $(\mathrm{d})=-4$

Fifth term $=-12-4=-16$ Sixth term $=-16-4=-20$

Seventh term $=-20-4=-24$

Therefore, next three terms are $-16,-20$ and -24

(viii) $-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\cdots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow -\frac{1}{2}-\left(-\frac{1}{2}\right)=-\frac{1}{2}-\left(-\frac{1}{2}\right)=0$

Common difference $(d)=0$

Fifth term $=-\frac{1}{2}+0=-\frac{1}{2}$ Sixth term $=-\frac{1}{2}+0=-\frac{1}{2}$

$\text{Seventh term } =-\frac{1}{2}+0=-\frac{1}{2}$

Therefore, next three terms are $-\frac{1}{2},-\frac{1}{2}$ and $-\frac{1}{2}$

(ix) $1,3,9,27\ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3-1 \neq 9-3$

(x) $a, 2a, 3a, 4a\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2a-a=3a-2a=4a-3a=a$

Common difference $(\mathrm{d})=\mathrm{a}$

Fifth term $=4a+a=5a$ Sixth term $=5a+a=6a$

Seventh term $=6a+a=7a$

Therefore, next three terms are $5a, 6a$ and $7a$

(xi) $a, a^{2}, a^{3}, a^{4}\ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow a^{2}-a \neq a^{3}-a^{2}$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}\ldots$

$\Rightarrow \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2\sqrt{2}-\sqrt{2}=3\sqrt{2}-2\sqrt{2}=\sqrt{2}$

Common difference $(\mathrm{d})=\sqrt{2}$

Fifth term $=4\sqrt{2}+\sqrt{2}=5\sqrt{2}$ Sixth term $=5\sqrt{2}+\sqrt{2}=6\sqrt{2}$

Seventh term $=6\sqrt{2}+\sqrt{2}=7\sqrt{2}$

Therefore, next three terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}\ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \sqrt{6}-\sqrt{3} \neq \sqrt{9}-\sqrt{6}$

$\text{(xiv) } 1^{2}, 3^{2}, 5^{2}, 7^{2}\ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3^{2}-1^{2} \neq 5^{2}-3^{2}$

$\text{(xv) } 1^{2}, 5^{2}, 7^{2}, 73\ldots$

$\Rightarrow 1, 25, 49, 73\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 5^2 - 1^2$

$= 7^2 - 5^2 = 73 - 7^2 = 24$

Common difference $(d) = 24$

Fifth term $= 73 + 24 = 97$ Sixth term $= 97 + 24 = 121$

Seventh term $= 121 + 24 = 145$

Therefore, next three terms are 97, 121 and 145

NCERT Solutions Class 10 Maths Chapter 5: Arithmetic Equations

Ex 5.2 Question 1.

Find the missing variable from a, d, n and a_n, where a is the first term, d is the common difference and a_n is the nth term of AP.
(i) $\mathrm{a}=7,\ \mathrm{d}=3, \mathrm{n}=8$
(ii) $\mathbf{a}=-18, \mathbf{n}=10, a_n=0$
(iii) $\mathbf{d}=-3, \mathbf{n}=18, a_n=-5$
(iv) $\mathbf{a}=-18.9, \mathbf{d}=2.5, a_n=3.6$
(v) $\mathbf{a}=3.5,\ \mathrm{d}=\mathbf{0}, \mathrm{n}=105$

Answer.

(i) $a=7, d=3, n=8$
We need to find a_n here.
Using formula $a_n=a+(n-1)d$
Putting values of a, d and n,
$a_n=7+(8-1)3$
$=7+(7)3=7+21=28$
(ii) $\mathrm{a}=-18, \mathrm{n}=10, a_n=0$

We need to find d here.
Using formula $a_n=a+(n-1)d$
Putting values of a, a_n and n,

$0=-18+(10-1)\mathrm{d}$
$\Rightarrow 0=-18+9d$
$\Rightarrow 18=9d \Rightarrow d=2$
(iii) $\mathrm{d}=-3, \mathrm{n}=18, a_n=-5$

We need to find a here.
Using formula $a_n=a+(n-1)d$
Putting values of d, a_n and n,
$-5=a+(18-1)(-3)$
$\Rightarrow -5=a+(17)(-3)$
$\Rightarrow -5=a-51 \Rightarrow a=46$
(iv) $\mathrm{a}=-18.9,\ \mathrm{d}=2.5, a_n=3.6$

We need to find n here.
Using formula $a_n=a+(n-1)d$
Putting values of d, a_n and n,
$3.6=-18.9+(n-1)(2.5)$
$\Rightarrow 3.6=-18.9+2.5n-2.5$
$\Rightarrow 2.5n=25 \Rightarrow n=10$
(v) $\mathrm{a}=3.5,\ \mathrm{d}=0, \mathrm{n}=105$

We need to find a_n here.

Using formula $a_n = a + (n-1)d$

Putting values of d, n and a,

$a_n = 3.5 + (105-1)(0)$

$\Rightarrow a_n = 3.5 + 104 \times 0$

$\Rightarrow a_n = 3.5 + 0 \Rightarrow a_n = 3.5$

Ex 5.2 Question 2.

Choose the correct choice in the following and justify:

(i) 30^{th} term of the AP: $10, 7, 4 \ldots$ is

(A) 97

(B) 77

(C) -77

(D) -87

(ii) 11^{th} term of the AP: $-3, -12, 2 \ldots$ is

(A) 28

(B) 22

(C) -38

(D) $-48\frac{1}{2}$

Answer.

(i) $10, 7, 4 \ldots$

First term $= a = 10$, Common difference $= d = 7 - 10 = 4 - 7 = -3$

And $n = 30 \{$ Because, we need to find 30^{th} term $\}$

$a_n = a + (n-1)d$

$\Rightarrow a_{30} = 10 + (30-1)(-3) = 10 - 87 = -77$

Therefore, the answer is (C).

(ii) $-3, -1/2, 2 \ldots$

First term $= \mathrm{a} = -3$, Common difference $= \mathrm{d} = -\frac{1}{2} - (-3) = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$

And $n = 11$ (Because, we need to find 11^{th} term)

$a_n = -3 + (11-1)\frac{5}{2} = -3 + 25 = 22$

Therefore 11^{th} term is 22 which means answer is (B).

Ex 5.2 Question 3.

In the following AP's find the missing terms:

(i) $2, \ldots, 26$

(ii) $, 13, \ldots, 3$

(iii) $5, \ldots, 9\frac{1}{2}$

(iv) $-4., \rightarrow, \rightarrow, 6$

(v) $, 38, \ldots, \ldots, -22$

Answer.

(i) $2, \ldots, 26$

We know that difference between consecutive terms is equal in any A.P.

Let the missing term be x.

$x - 2 = 26 - x$

$\Rightarrow 2x = 28 \Rightarrow x = 14$

Therefore, missing term is 14 .

(ii) $, 13, \ldots, 3$

Let missing terms be x and y.

The sequence becomes $x, 13, y, 3$

We know that difference between consecutive terms is constant in any A.P.

$y - 13 = 3 - y$

$\Rightarrow 2y = 16 \Rightarrow y = 8$

And $13 - x = y - 13$

$\Rightarrow x + y = 26$

But, we have $y = 8$,

$\Rightarrow x + 8 = 26 \Rightarrow x = 18$

Therefore, missing terms are 18 and 8 .

(iii) $5, \ldots, 9\frac{1}{2}$

Here, first term $= a = 5$ And, 4^{th} term $= a_4 = 9\frac{1}{2}$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a_4 = 5 + (4-1)d$
$\Rightarrow \frac{19}{2} = 5 + 3d$
$\Rightarrow 19 = 2(5 + 3d)$
$\Rightarrow 19 = 10 + 6\text{ d}$
$\Rightarrow 6\text{ d} = 19 - 10$
$\Rightarrow 6\text{ d} = 9 \Rightarrow d = \frac{3}{2}$

Therefore, we get common difference $= d = \frac{3}{2}$
Second term $= a + d = 5 + \frac{3}{2} = \frac{13}{2}$
Third term $=$ second term $+\text{d} = \frac{13}{2} + \frac{3}{2} = \frac{16}{2} = 8$
Therefore, missing terms are $\frac{13}{2}$ and 8
(iv) $-4., \rightarrow, \rightarrow, 6$

Here, First term $=$ a $= -4$ and 6^{th} term $= a_6 = 6$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a_6 = -4 + (6-1)d$
$\Rightarrow 6 = -4 + 5d$
$\Rightarrow 5d = 10 \Rightarrow d = 2$

Therefore, common difference $= d = 2$
Second term $=$ first term $+d = a + d = -4 + 2 = -2$
Third term $=$ second term $+d = -2 + 2 = 0$
Fourth term $=$ third term $+d = 0 + 2 = 2$
Fifth term $=$ fourth term $+d = 2 + 2 = 4$

Therefore, missing terms are $-2, 0, 2$ and 4 .
(v) $, 38, \ldots, \rightarrow, -22$

We are given 2^{nd} and 6^{th} term.
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression, $a_2 = a + (2-1)d$ and $a_6 = a + (6-1)d$
$\Rightarrow 38 = a + d$ and $-22 = a + 5d$

These are equations in two variables, we can solve them using any method.
Using equation ($38 = a + d$), we can say that $a = 38 - d$.
Putting value of a in equation $(-22 = a + 5d)$,
$-22 = 38 - d + 5d$
$\Rightarrow 4d = -60$
$\Rightarrow d = -15$

Using this value of d and putting this in equation $38 = a + d$,
$38 = a - 15 \Rightarrow a = 53$

Therefore, we get $a = 53$ and $d = -15$
First term $= a = 53$
Third term $=$ second term $+\text{d} = 38 - 15 = 23$
Fourth term $=$ third term $+d = 23 - 15 = 8$
Fifth term = fourth term $+d = 8 - 15 = -7$
Therefore, missing terms are 53, 23, 8 and -7 .

Ex 5.2 Question 4.

Which term of the AP: $3, 8, 13, 18 \ldots$ is 78?

Answer.

First term $= a = 3$, Common difference $= \text{d} = 8 - 3 = 13 - 8 = 5$ and $a_n = 78$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a_n = 3 + (n-1)5,$
$\Rightarrow 78 = 3 + (n-1)5$
$\Rightarrow 75 = 5n - 5$
$\Rightarrow 80 = 5n \Rightarrow n = 16$
It means 16^{th} term of the given AP is equal to 78 .

Ex 5.2 Question 5.

Find the number of terms in each of the following APs:
(i) $7, 13, 19 \ldots, 205$
(ii) $18, 15\frac{1}{2}, 13 \ldots, -47$

Answer.

(i) $7, 13, 19 \ldots, 205$
First term $=$ a $= 7$, Common difference $=$ d $= 13 - 7 = 19 - 13 = 6$
And $a_n = 205$
Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, $205 = 7 + (n-1)6 = 7 + 6n - 6$
$\Rightarrow 205 = 6n + 1$
$\Rightarrow 204 = 6n \Rightarrow n = 34$
Therefore, there are 34 terms in the given arithmetic progression.
(ii) $18, 15\frac{1}{2}, 13 \ldots, -47$

First term $=$ a $= 18$, Common difference d $= 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{31-36}{2} = \frac{-5}{2}$
And $a_n = -47$
Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$
$$= 36 - \frac{5}{2}n + \frac{5}{2}$$
$$\Rightarrow -94 = 36 - 5n + 5$$
$$\Rightarrow 5n = 135 \Rightarrow n = 27$$

Therefore, there are 27 terms in the given arithmetic progression.

Ex 5.2 Question 6.

Check whether -150 is a term of the AP: $11, 8, 5, 2 \ldots$

Answer.

Let -150 is the n^{th} of AP $11, 8, 5, 2 \ldots$ which means that $a_n = -150$
Here, First term $=$ a $= 11$, Common difference $=$ d $= 8 - 11 = -3$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$$-150 = 11 + (n-1)(-3)$$
$$\Rightarrow -150 = 11 - 3n + 3$$
$$\Rightarrow 3n = 164 \Rightarrow n = \frac{164}{3}$$

But, n cannot be in fraction.
Therefore, our supposition is wrong. -150 cannot be term in AP.

Ex 5.2 Question 7.

Find the 31^{st} term of an AP whose 11^{th} term is 38 and 16^{th} term is 73.

Answer.

Here $a_{11} = 38$ and $a_{16} = 73$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression, $38 = a + (11-1)(d)$ And $73 = a + (16-1)(d)$
$\Rightarrow 38 = a + 10d$ And $73 = a + 15d$
These are equations consisting of two variables.
We have, $38 = a + 10d$
$\Rightarrow a = 38 - 10d$
Let us put value of a in equation $(73 = a + 15d)$,

$$73 = 38 - 10d + 15d$$
$$\Rightarrow 35 = 5d$$

Therefore, Common difference $= d = 7$
Putting value of d in equation $38 = a + 10d$,

$$38 = a + 70$$
$$\Rightarrow a = -32$$

Therefore, common difference $= d = 7$ and First term $= a = -32$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$$a_{31} = -32 + (31-1)(7)$$
$$= -32 + 210 = 178$$

Therefore, 31^{st} term of AP is 178.

Ex 5.2 Question 8.

An AP consists of 50 terms of which 3^{rd} term is 12 and the last term is 106 . Find the 29^{th} term.

Answer.

An AP consists of 50 terms and the 50^{th} term is equal to 106 and $a_3 = 12$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$$a_{50} = a + (50-1)d \text{ And } a_3 = a + (3-1)d$$
$$\Rightarrow 106 = a + 49d \text{ And} 12 = a + 2d$$

These are equations consisting of two variables.
Using equation $106=a+49d$, we get $a=106-49d$
Putting value of a in the equation $12=a+2d$,
$12=106-49d+2d$
$\Rightarrow 47d=94 \Rightarrow d=2$

Putting value of d in the equation, $a=106-49d$,
$a=106-49(2)=106-98=8$

Therefore, First term $=a=8$ and Common difference $=d=2$
To find 29^{th} term, we use formula $a_n=a+(n-1)d$ which is used to find n^{th} term of arithmetic progression,
$a_{29}=8+(29-1)2=8+56=64$

Therefore, 29th term of AP is equal to 64 .

Ex 5.2 Question 9.

If the third and the ninth terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Answer.

It is given that 3^{rd} and 9^{th} term of AP are 4 and -8 respectively.

It means $a_3=4$ and $a_9=-8$
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression, $4=a+(3-1)d$ And, $-8=a+(9-1)d$
$\Rightarrow 4=a+2d$ and $-8=a+8d$
These are equations in two variables.
Using equation $4=a+2d$, we can say that $a=4-2d$
Putting value of a in other equation $-8=a+8d$,
$-8=4-2d+8d$
$\Rightarrow -12=6d \Rightarrow d=-2$

Putting value of d in equation $-8=a+8d$,
$-8=a+8(-2)$
$\Rightarrow -8=a-16 \Rightarrow a=8$

Therefore, first term $=a=8$ and Common Difference $=d=-2$
We want to know which term is equal to zero.
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$0=8+(n-1)(-2)$
$\Rightarrow 0=8-2n+2$
$\Rightarrow 0=10-2n$
$\Rightarrow 2n=10 \Rightarrow n=5$

Therefore, 5^{th} term is equal to 0.

Ex 5.2 Question 10.

The 17^{th} term of an AP exceeds its 10^{th} term by 7 . Find the common difference.

Answer.

$a_{17}=a_{10}+7\ldots$
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression, $a_{17}=a+16d\ldots(2)$
$a_{10}=a+9d\ldots$

Putting (2) and (3) in equation (1),
$a+16d=a+9d+7$
$\Rightarrow 7d=7$
$\Rightarrow d=1$

Ex 5.2 Question 11.

Which term of the AP: $3,15,27,39\ldots$ will be 132 more than its 54^{th} term?

Answer.

Lets first calculate 54^{th} of the given AP.
First term $=a=3$, Common difference $=d=15-3=12$
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$a_{54}=a+(54-1)d=3+53(12)=3+636=639$

We want to find which term is 132 more than its 54^{th} term.
Let us suppose it is n^{th} term which is 132 more than 54^{th} term.
$a_n=a_{54}+132$
$\Rightarrow 3+(n-1)12=639+132$

$\Rightarrow 3+12n-12=771$

$\Rightarrow 12n - 9 = 771$
$\Rightarrow 12n = 780$
$\Rightarrow n = 65$

Therefore, 65^{th} term is 132 more than its 54^{th} term.

Ex 5.2 Question 12.

Two AP's have the same common difference. The difference between their 100^{th} terms is 100 , what is the difference between their 1000^{th} terms.

Answer.

Let first term of 1^{st} AP $= a$

Let first term of 2^{nd} AP $= a'$
It is given that their common difference is same.
Let their common difference be d.

It is given that difference between their 100^{th} terms is 100 .

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a + (100-1)d - [a' + (100-1)d]$
$= a + 99d - a' - 99d = 100$
$\Rightarrow a - a' = 100 \ldots (1)$

We want to find difference between their 1000^{th} terms which means we want to calculate:
$a + (1000-1)d - [a' + (1000-1)d]$
$= a + 999d - a' - 999d = a - a'$

Putting equation (1) in the above equation,
$a + (1000-1)d - [a' + (1000-1)d]$

$= a + 999d - a' + 999d = a - a' = 100$

Therefore, difference between their 1000^{th} terms would be equal to 100 .

Ex 5.2 Question 13.

How many three digit numbers are divisible by 7 ?

Answer.

We have AP starting from 105 because it is the first three digit number divisible by 7. AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have AP of the form 105, 112, 119..., 994

Let 994 is the n^{th} term of AP.
We need to find n here.

First term $= \text{a} = 105$, Common difference $= \text{d} = 112 - 105 = 7$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$994 = 105 + (n-1)(7)$
$\Rightarrow 994 = 105 + 7n - 7$
$\Rightarrow 896 = 7n \Rightarrow n = 128$

It means 994 is the 128^{th} term of AP.
Therefore, there are 128 terms in AP.

Ex 5.2 Question 14.

How many multiples of 4 lie between 10 and 250 ?

Answer.

First multiple of 4 which lie between 10 and 250 is 12.
The last multiple of 4 which lie between 10 and 250 is 248 .
Therefore, AP is of the form $12, 16, 20 \ldots, 248$
First term $= \text{a} = 12$, Common difference $= \text{d} = 4$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$248 = 12 + (n-1)(4)$
$\Rightarrow 248 = 12 + 4n - 4$
$\Rightarrow 240 = 4n$
$\Rightarrow n = 60$

It means that 248 is the 60^{th} term of AP.

So, we can say that there are 60 multiples of 4 which lie between 10 and 250 .

Ex 5.2 Question 15.

For what value of n, are the nth terms of two AP's: $63, 65, 67 \ldots$ and $3, 10, 17 \ldots$ equal?

Answer.

Lets first consider AP $63, 65, 67 \ldots$
First term $= a = 63$, Common difference $= d = 65 - 63 = 2$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a_n = 63 + (n-1)(2) \ldots (1)$

Now, consider second AP $3, 10, 17 \ldots$
First term $= a = 3$, Common difference $= d = 10 - 3 = 7$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a_n = 3 + (n-1)(7) \ldots (2)$

According to the given condition:
$(1) = (2)$

$\Rightarrow 63 + (n-1)(2) = 3 + (n-1)(7)$
$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$

$\Rightarrow 65 = 5n \Rightarrow n = 13$

Therefore, 13^{th} terms of both the AP's are equal.

Ex 5.2 Question 16.

Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Answer.
Let first term of $\mathrm{AP} = \mathrm{a}$

Let common difference of $\mathrm{AP} = \mathrm{d}$

It is given that its 3^{rd} term is equal to 16 .
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$16 = a + (3-1)(d)$
$\Rightarrow 16 = a + 2d \ldots (1)$

It is also given that 7^{th} term exceeds 5^{th} term by12.
According to the given condition:
$a_7 = a_5 + 12$
$\Rightarrow a + (7-1)d = a + (5-1)d + 12$
$\Rightarrow 2d = 12 \Rightarrow d = 6$

Putting value of d in equation $16 = a + 2d$,
$16 = a + 2(6) \Rightarrow a = 4$

Therefore, first term $= a = 4$
And, common difference $= d = 6$
Therefore, AP is $4, 10, 16, 22 \ldots$

Ex 5.2 Question 17.

Find the 20^{th} term from the last term of the AP: 3, 8, 13..., 253.

Answer.

We want to find 20^{th} term from the last term of given AP.
So, let us write given AP in this way: $253 \ldots 13, 8, 3$
Here First term $= a = 253$, Common Difference $= d = 8 - 13 = -5$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a_{20} = 253 + (20-1)(-5)$
$\Rightarrow a_{20} = 253 + 19(-5) = 253 - 95 = 158$

Therefore, the 20^{th} term from the last term of given AP is 158 .

Ex 5.2 Question 18.

The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of 6^{th} and 10^{th} terms is 44. Find the three terms of the AP.

Answer.

The sum of 4^{th} and 8^{th} terms of an AP is 24 and sum of 6^{th} and 10^{th} terms is 44 .
$a_4 + a_8 = 24$
$\text{and } a_6 + a_{10} = 44$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$\Rightarrow a + (4-1)d + [a + (8-1)d] = 24$

And, $a + (6-1)d + [a + (10-1)d] = 44$
$\Rightarrow a + 3d + a + 7d = 24$

And $a+5d+a+9d=44$
$\Rightarrow 2a+10d=24$ And $2a+14d=44$
$\Rightarrow a+5d=12$ And $a+7d=22$

These are equations in two variables.
Using equation, $a+5d=12$, we can say that $a=12-5d\ldots$ (1)
Putting (1) in equation $a+7d=22$,
$12-5d+7d=22$
$\Rightarrow 12+2d=22$
$\Rightarrow 2d=10$
$\Rightarrow d=5$

Putting value of d in equation $a=12-5d$,
$a=12-5(5)=12-25=-13$

Therefore, first term $=a=-13$ and, Common difference $=d=5$
Therefore, AP is $-13,-8,-3,2\ldots$
Its first three terms are $-13,-8$ and -3 .

Ex 5.2 Question 19.

Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Answer.

Subba Rao's starting salary = Rs 5000
It means, first term $=a=5000$
He gets an increment of Rs 200 after every year.
Therefore, common difference $=d=200$
His salary after 1 year $=5000+200=$ Rs 5200
His salary after two years $=5200+200=$ Rs 5400
Therefore, it is an AP of the form $5000,5200,5400,5600\ldots,7000$

We want to know in which year his income reaches Rs 7000.
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$7000=5000+(n-1)(200)$
$\Rightarrow 7000=5000+200n-200$
$\Rightarrow 7000-5000+200=200n$
$\Rightarrow 2200=200n$
$\Rightarrow n=11$

It means after 11 years, Subba Rao's income would be Rs 7000.

Ex 5.2 Question 20.

Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs 20.75 , find n.

Answer.

Ramkali saved Rs. 5 in the first week of year. It means first term $=\mathrm{a}=5$
Ramkali increased her weekly savings by Rs 1.75 .
Therefore, common difference $=\mathrm{d}=$ Rs 1.75
Money saved by Ramkali in the second week $=\mathrm{a}+\mathrm{d}=5+1.75=$ Rs 6.75
Money saved by Ramkali in the third week $=6.75+1.75=$ Rs 8.5
Therefore, it is an AP of the form:5, 6.75, $8.5\ldots,20.75$
We want to know in which week her savings become 20.75 .
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$20.75=5+(n-1)(1.75)$
$\Rightarrow 20.75=5+1.75n-1.75$
$\Rightarrow 17.5=1.75n$
$\Rightarrow n=10$

It means in the 10^{th} week her savings become Rs 20.75 .

NCERT Solutions Class 10 Maths Chapter 5: Arithmetic Equations

Ex 5.3 Question 1.

Find the sum of the following AP's.
(i) $2, 7, 12 \ldots$ to 10 terms
(ii) $-37, -33, -29 \ldots$ to 12 terms
(iii) $0.6, 1.7, 2.8 \ldots$ to 100 terms
(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ to 11 terms

Answer.

(i) $2, 7, 12 \ldots$ to 10 terms
Here First term $= a = 2$, Common difference $= d = 7 - 2 = 5$ and $n = 10$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,
$S_n = \frac{10}{2}[4 + (10-1)5] = 5(4 + 45) = 5 \times 49 = 245$
(ii) $-37, -33, -29 \ldots$ to 12 terms

Here First term $= a = -37$, Common difference $= d = -33 - (-37) = 4$
And $n = 12$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,
$S_n = \frac{12}{2}[-74 + (12-1)4] = 6(-74 + 44) = 6 \times (-30) = -180$

(iii) $0.6, 1.7, 2.8 \ldots$ to 100 terms

Here First term $= \mathrm{a} = 0.6$, Common difference $= \mathrm{d} = 1.7 - 0.6 = 1.1$
And $n = 100$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,
$Sn = \frac{100}{2}[1.2 + (100-1)1.1] = 50(1.2 + 108.9) = 50 \times 110.1 = 5505$
(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ to 11 terms

Here First term $= \mathrm{a} = \frac{1}{15}$ Common difference $= \mathrm{d} = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$
Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,
$S_n = \frac{11}{2}\left[\frac{2}{15} + (11-1)\frac{1}{60}\right] = \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right) = \frac{11}{2}\left(\frac{4+5}{30}\right) = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$

Ex 5.3 Question 2.

Find the sums given below:
(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$
(ii) $34 + 32 + 30 + \ldots + 10$
(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Answer.

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

Here First term $= \mathrm{a} = 7$, Common difference $= \mathrm{d} = \frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2} = 3.5$
And Last term $= l = 84$

We do not know how many terms are there in the given AP.
So, we need to find n first.
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$[7 + (n-1)(3.5)] = 84$

$\Rightarrow 7 + (3.5)n - 3.5 = 84$

$\Rightarrow 3.5n = 84 + 3.5 - 7$

$\Rightarrow 3.5n = 80.5$

$\Rightarrow n = 23$

Therefore, there are 23 terms in the given AP.
It means $\mathrm{n} = 23$
Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{23} = \frac{23}{2}(7 + 84)$$

$$\Rightarrow S_{23} = \frac{23}{2} \times 91 = 1046.5$$

(ii) $34 + 32 + 30 + \ldots + 10$

Here First term $= \mathrm{a} = 34$, Common difference $= \mathrm{d} = 32 - 34 = -2$
And Last term $= l = 10$
We do not know how many terms are there in the given AP.

So, we need to find n first.
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$[34 + (n-1)(-2)] = 10$

$\Rightarrow 34 - 2n + 2 = 10$

$\Rightarrow -2n = -26 \Rightarrow n = 13$

Therefore, there are 13 terms in the given AP.
It means $\mathrm{n} = 13$.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,
$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2} \times 44 = 286$
(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Here First term $= \mathrm{a} = -5$, Common difference $= \mathrm{d} = -8 - (-5) = -8 + 5 = -3$
And Last term $= l = -230$
We do not know how many terms are there in the given AP.
So, we need to find n first.
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$[-5 + (n-1)(-3)] = -230$

$\Rightarrow -5 - 3n + 3 = -230$

$\Rightarrow -3n = -228 \Rightarrow n = 76$

Therefore, there are 76 terms in the given AP.
It means $\mathrm{n} = 76$.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$S_{76} = \frac{76}{2}(-5 - 230) = 38 \times (-235) = -8930$

Ex 5.3 Question 3.

In an AP
(i) given $a = 5, \mathbf{d} = 3, a_n = 50$, find n and S_n.
(ii) given $a = 7, a_{13} = 35$, find d and S_{13}.
(iii) given $a_{12} = 37, \mathbf{d} = 3$, find a and S_{12}.
(iv) given $a_3 = 15, S_{10} = 125$, find $\mathbf{d}$ and a_{10}.
(v) given $d = 5, S_9 = 75$, find a and a_9.
(vi) given $a = 2, d = 8, S_n = 90$, findn and a_n.
(vii) given $a = 8, a_n = 62, S_n = 210$, find n and d.
(viii) given $a_n = 4, \mathbf{d} = 2, S_n = -14$, find n and a.
(ix) given $a = 3, n = 8, S = 192$, find d.
(x) given $l = 28, S = 144$, and there are total of 9 terms. Find a.

Answer.

(i) Given $a = 5, d = 3, a_n = 50$, find n and S_n.
Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_n = 5 + (n - 1)(3)$

$\Rightarrow 50 = 5 + 3n - 3$

$\Rightarrow 48 = 3n \Rightarrow n = 16$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP,

$S_{16} = \frac{16}{2}[10 + (16 - 1)3] = 8(10 + 45) = 8 \times 55 = 440$

Therefore, $n = 16$ and $S_n = 440$
(ii) Given $a = 7, a_{13} = 35$, find d and S_{13}.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_{13} = 7 + (13 - 1)(d)$

$\Rightarrow 35 = 7 + 12d$

$\Rightarrow 28 = 12d \Rightarrow d = \frac{7}{3}$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP,

$S_{13} = \frac{13}{2}\left[14 + (13 - 1)\frac{7}{3}\right] = \frac{13}{2}(14 + 28) = \frac{13}{2} \times 42 = 273$

Therefore, $d = \frac{7}{3}$ and $S_{13} = 273$
(iii) Given $a_{12} = 37, d = 3$, find a and S_{12}.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_{12} = a + (12 - 1)3$

$\Rightarrow 37 = a + 33 \Rightarrow a = 4$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP,

$S_{12} = \frac{12}{2}[8 + (12 - 1)3] = 6(8 + 33) = 6 \times 41 = 246$

Therefore, $a = 4$ and $S_{12} = 246$
(iv) Given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_3 = a + (3 - 1)(d)$

$\Rightarrow 15 = a + 2d$

$\Rightarrow a = 15 - 2d \ldots (1)$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP,

$S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

$\Rightarrow 125 = 5(2a + 9d) = 10a + 45d$

Putting (1) in the above equation,

$125 = 5[2(15 - 2d) + 9d] = 5(30 - 4d + 9d)$

$\Rightarrow 125 = 150 + 25d$

$\Rightarrow 125 - 150 = 25d$

$\Rightarrow -25 = 25d \Rightarrow d = -1$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_{10} = a + (10 - 1)d$

Putting value of d and equation (1) in the above equation,

$a_{10} = 15 - 2d + 9d = 15 + 7d$

$= 15 + 7(-1) = 15 - 7 = 8$

Therefore, $d = -1$ and $a_{10} = 8$
(v) Given $d = 5, S_9 = 75$, find a and a_9.

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP,

$S_9 = \frac{9}{2}[2a + (9 - 1)5]$

$\Rightarrow 75 = \frac{9}{2}[2a + 40]$

$\Rightarrow 150 = 18a + 360$

$\Rightarrow -210 = 18a$

$\Rightarrow a = \frac{-35}{3}$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_9 = \frac{-35}{3} + (9 - 1)(5)$

$= \frac{-35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$

Therefore, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$
(vi) Given $a = 2, d = 8, S_n = 90$, find n and a_n.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$\Rightarrow 90 = \frac{n}{2}[4 + 8n - 8]$$
$$\Rightarrow 90 = \frac{n}{2}[8n - 4]$$
$$\Rightarrow 8n^2 - 4n - 180 = 0$$
$$\Rightarrow 2n^2 - n - 45 = 0$$
$$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$$
$$\Rightarrow 2n(n-5) + 9(n-5) = 0$$
$$\Rightarrow (n-5)(2n+9) = 0$$
$$\Rightarrow n = 5, -9/2$$

We discard negative value of n because here n cannot be in negative or fraction.
The value of n must be a positive integer.
Therefore, $n = 5$
Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$a_5 = 2 + (5-1)(8) = 2 + 32 = 34$

Therefore, $n = 5$ and $a_n = 34$
(vii) Given $a = 8, a_n = 62, S_n = 210$, find n and d.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$62 = 8 + (n-1)(d) = 8 + nd - d$

$$\Rightarrow 62 = 8 + nd - d$$
$$\Rightarrow nd - d = 54$$

$$\Rightarrow nd = 54 + d \ldots (1)$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,
$210 = \frac{n}{2}[16 + (n-1)d] = \frac{n}{2}(16 + nd - d)$

Putting equation (1) in the above equation,
$$210 = \frac{n}{2}[16 + 54 + d - d] = \frac{n}{2} \times 70$$
$$\Rightarrow n = \frac{210 \times 2}{70} = 6 \Rightarrow n = 6$$

Putting value of n in equation (1),
$6d = 54 + d \Rightarrow d = \frac{54}{5}$

Therefore, $n = 6$ and $d = \frac{54}{5}$
(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
$$4 = a + (n-1)(2) = a + 2n - 2$$
$$\Rightarrow 4 = a + 2n - 2$$
$$\Rightarrow 6 = a + 2n$$
$$\Rightarrow a = 6 - 2n \ldots (1)$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$-14 = \frac{n}{2}[2a + (n-1)2] = \frac{n}{2}(2a + 2n - 2)$$
$$\Rightarrow -14 = \frac{n}{2}(2a + 2n - 2)$$

Putting equation (1) in the above equation, we get
$$-28 = n[2(6-2n) + 2n - 2]$$
$$\Rightarrow -28 = n(12 - 4n + 2n - 2)$$
$$\Rightarrow -28 = n(10 - 2n)$$
$$\Rightarrow 2n^2 - 10n - 28 = 0$$
$$\Rightarrow n^2 - 5n - 14 = 0$$
$$\Rightarrow n^2 - 7n + 2n - 14 = 0$$
$$\Rightarrow n(n-7) + 2(n-7) = 0$$
$$\Rightarrow (n+2)(n-7) = 0$$
$$\Rightarrow n = -2, 7$$

Here, we cannot have negative value of n.
Therefore, we discard negative value of n which means $n = 7$.
Putting value of n in equation (1), we get
$a = 6 - 2n = 6 - 2(7) = 6 - 14 = -8$

Therefore, $n=7$ and $a=-8$
(ix)Given $a=3, n=8, S=192$, find d.

Using formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$$192=\frac{8}{2}[6+(8-1)d]=4(6+7d)$$
$$\Rightarrow 192=24+28d$$
$$\Rightarrow 168=28d \Rightarrow d=6$$

(x) Given $l=28, S=144$, and there are total of 9 terms. Find a.

Applying formula, $S_n=\frac{n}{2}[a+l]$, to find sum of n terms, we get

$$144=\frac{9}{2}[a+28]$$
$$\Rightarrow 288=9[a+28]$$
$$\Rightarrow 32=a+28 \Rightarrow a=4$$

Ex 5.3 Question 4.

How many terms of the AP: $9,17,25,\ldots$ must be taken to give a sum of 636?

Answer.

First term $=$ a $=9$, Common difference $=$ d $=17-9=8, S_n=636$

Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$$636=\frac{n}{2}[18+(n-1)(8)]$$
$$\Rightarrow 1272=n(18+8n-8)$$
$$\Rightarrow 1272=18n+8n^2-8n$$
$$\Rightarrow 8n^2+10n-1272=0$$
$$\Rightarrow 4n^2+5n-636=0$$

Comparing equation $4n^2+5n-636=0$ with general form $cn^2+bn+c=0$, we get

$a=4, b=5$ and $c=-636$

Applying quadratic formula, $n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and putting values of a, b and c, we get

$$n=\frac{-5\pm\sqrt{5^2-4(4)(-636)}}{8}$$
$$\Rightarrow n=\frac{-5\pm\sqrt{25+10176}}{8}$$
$$\Rightarrow n=\frac{-5\pm 101}{8}$$
$$\Rightarrow n=\frac{96}{8},\frac{-106}{8}=12,-\frac{106}{8}$$

We discard negative value of n here because n cannot be in negative, n can only be a positive integer.

Therefore, $n=12$
Therefore, 12 terms of the given sequence make sum equal to 636 .

Ex 5.3 Question 5.

The first term of an AP is 5 , the last term is 45 and the sum is 400 . Find the number of terms and the common difference.

Answer.

First term $=$ a $=5$, Last term $=l=45, S_n=400$
Applying formula, $S_n=\frac{n}{2}[a+l]$ to find sum of n terms of AP, we get

$$400=\frac{n}{2}[5+45]$$
$$\Rightarrow \frac{400}{50}=\frac{n}{2}\Rightarrow \mathrm{n}=16$$

Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP and putting value of

n, we get

$$400=\frac{16}{2}[10+(16-1)d]$$
$$\Rightarrow 400=8(10+15d)$$
$$\Rightarrow 400=80+120d$$
$$\Rightarrow 320=120d$$
$$\Rightarrow d=\frac{320}{120}=\frac{8}{3}$$

Ex 5.3 Question 6.

The first and the last terms of an AP are 17 and 350 respectively. If, the common difference is 9 , how many terms are there and what is their sum?

Answer.

First term $= a = 17$, Last term $= l = 350$ and Common difference $= d = 9$
Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$350 = 17 + (n-1)(9)$
$\Rightarrow 350 = 17 + 9n - 9$
$\Rightarrow 342 = 9n \Rightarrow n = 38$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of n, we get

$S_{38} = \frac{38}{2}[34 + (38-1)\theta]$
$\Rightarrow S_{38} = 19(34 + 333) = 6973$

Therefore, there are 38 terms and their sum is equal to 6973.

Ex 5.3 Question 7.

Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149 .

Answer.

It is given that 22nd term is equal to $149 \Rightarrow a_{22} = 149$
Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$149 = a + (22-1)(7)$
$\Rightarrow 149 = a + 147 \Rightarrow a = 2$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of a, we get

$S_{22} = \frac{22}{2}[4 + (22-1)7]$
$\Rightarrow S_{22} = 11(4 + 147)$
$\Rightarrow S_{22} = 1661$

Therefore, sum of first 22 terms of AP is equal to 1661.

Ex 5.3 Question 8.

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer.

It is given that second and third term of AP are 14 and 18 respectively.
Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$14 = a + (2-1)d$
$\Rightarrow 14 = a + d \ldots (1)$

And, $18 = a + (3-1)d$

$\Rightarrow 18 = a + 2d \ldots (2)$

These are equations consisting of two variables.

Using equation (1), we get, $a = 14 - d$
Putting value of a in equation (2), we get

$18 = 14 - d + 2d$
$\Rightarrow d = 4$

Therefore, common difference $d = 4$
Putting value of d in equation (1), we get

$18 = a + 2\ (4)$
$\Rightarrow a = 10$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get
$S_{51} = \frac{51}{2}[20 + (51-1)d] = \frac{51}{2}(20 + 200) = \frac{51}{2} \times 220 = 51 \times 110 = 5610$

Therefore, sum of first 51 terms of an AP is equal to 5610 .

Ex 5.3 Question 9.

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289 , find the sum of first n terms.

Answer.

It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get
$49 = \frac{7}{2}[2a + (7-1)d]49$

$\Rightarrow 98 = 7(2a + 6d)$
$\Rightarrow 7 = a + 3d \Rightarrow a = 7 - 3d..$

And, $289 = \frac{17}{2}[2a + (17-1)d]$
$\Rightarrow 578 = 17(2a + 16d)$
$\Rightarrow 34 = 2a + 16d$
$\Rightarrow 17 = a + 8d$

Putting equation (1) in the above equation, we get

$17=7-3d+8d$

$\Rightarrow 10=5d \Rightarrow d=2$

Putting value of d in equation (1), we get

$a=7-3d=7-3(2)=7-6=1$

Again applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_n=\frac{n}{2}[2(1)+(n-1)2]$

$\Rightarrow S_n=\frac{n}{2}[2+2n-2]$

$\Rightarrow S_n=\frac{n}{2}\times 2n \Rightarrow S_n=n^2$

Therefore, sum of n terms of AP is equal to n^2.

Ex 5.3 Question 10.

Show that $a_1, a_2 \ldots a_n$ form an AP where a_n is defined as below:

(i) $a_n=3+4n$

(ii) $a_n=9-5n$

Also find the sum of the first 15 terms in each case.

Answer.

(i)We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n=3+4n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n=3+4n$

$a_1=3+4(1)=3+4=7$

$a_2=3+4(2)=3+8=11$

$a_3=3+4(3)=3+12=15$

$a_4=3+4(4)=3+16=19$

So, the sequence is of the form $7, 11, 15, 19 \ldots$

Let us check difference between consecutive terms of this sequence.

$11-7=4, 15-11=4, 19-15=4$

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence $7, 11, 15, 19 \ldots$

First term = a = 7 and Common difference $=d=4$

Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_{15}=\frac{15}{2}[14+(15-1)4]=\frac{15}{2}(14+56)=\frac{15}{2}\times 70=15\times 35=525$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii)We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n=9-5n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n=9-5n$

$a_1=9-5(1)=9-5=4 \quad a_2=9-5(2)=9-10=-1$

$a_3=9-5(3)=9-15=-6 \quad a_4=9-5(4)=9-20=-11$

So, the sequence is of the form $4, -1, -6, -11 \ldots$

Let us check difference between consecutive terms of this sequence.

$-1-(4)=-5, -6-(-1)$

$=-6+1=-5, -11-(-6)$

$=-11+6=-5$

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence $4, -1, -6, -11 \ldots$

First term $=a=4$ and Common difference $=d=-5$

Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_{15}=\frac{15}{2}[8+(15-1)(-5)]=\frac{15}{2}(8-70)=\frac{15}{2}\times(-62)=15\times(-31)=-465$

Therefore, sum of first 15 terms of AP is equal to -465 .

Ex 5.3 Question 11.

If the sum of the first n terms of an AP is $\left(4n-n^2\right)$, what is the first term (that is S_1) ? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Answer.

It is given that the sum of n terms of an AP is equal to $\left(4n-n^2\right)$

It means $S_n=4n-n^2$

Let us calculate S_1 and S_2 using $S_n=4n-n^2$

$S_1 = 4(1) - (1)^2 = 4 - 1 = 3$
$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$

First term $= \mathrm{a} = S_1 = 3$
Let us find common difference now.
We can write any AP in the form of general terms like $a, a+d, a+2d \ldots$
We have calculated that sum of first two terms is equal to 4 i.e. $S_2 = 4$
Therefore, we can say that $a + (a+d) = 4$
Putting value of a from equation (1), we get
$2a + d = 4$
$\Rightarrow 2(3) + d = 4$
$\Rightarrow 6 + d = 4$
$\Rightarrow d = -2$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,
Second term of $\mathrm{AP} = a_2 = \mathrm{a} + (2-1)\mathrm{d} = 3 + (2-1)(-2) = 3 - 2 = 1$
Third term of $\mathrm{AP} = a_3 = \mathrm{a} + (3-1)\mathrm{d} = 3 + (3-1)(-2) = 3 - 4 = -1$
Tenth term of AP $= a_{10} = \mathrm{a} + (10-1)\mathrm{d} = 3 + (10-1)(-2) = 3 - 18 = -15$
n^{th} term of $\mathrm{AP} = a_n = \mathrm{a} + (\mathrm{n}-1)\mathrm{d} = 3 + (\mathrm{n}-1)(-2) = 3 - 2\mathrm{n} + 2 = 5 - 2\mathrm{n}$

Ex 5.3 Question 12.

Find the sum of the first 40 positive integers divisible by 6 .

Answer.

The first 40 positive integers divisible by 6 are $6, 12, 18, 24 \ldots 40$ terms.
Therefore, we want to find sum of 40 terms of sequence of the form:

$6, 12, 18, 24 \ldots 40$ terms
Here, first term $= a = 6$ and Common difference $= d = 12 - 6 = 6, n = 40$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get
$S_{40} = \frac{40}{2}[12 + (40-1)6]$
$= 20(12 + 39 \times 6)$
$= 20(12 + 234)$
$= 20 \times 246 = 4920$

Ex 5.3 Question 13.

Find the sum of the first 15 multiples of 8.

Answer.

The first 15 multiples of 8 are $8, 16, 24, 32 \ldots 15$ terms
First term $= a = 8$ and Common difference $= d = 16 - 8 = 8, n = 15$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get
$S_{15} = \frac{15}{2}[16 + (15-1)8] = \frac{15}{2}(16 + 14 \times 8) = \frac{15}{2}(16 + 112) = \frac{15}{2} \times 128 = 15 \times 64 = 960$

Ex 5.3 Question 14.

Find the sum of the odd numbers between 0 and 50 .

Answer.

The odd numbers between 0 and 50 are $1, 3, 5, 7 \ldots 49$
It is an arithmetic progression because the difference between consecutive terms is constant.
First term $= \mathrm{a} = 1$, Common difference $= 3 - 1 = 2$, Last term $= l = 49$
We do not know how many odd numbers are present between 0 and 50 .
Therefore, we need to find n first.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get
$49 = 1 + (n-1)2$
$\Rightarrow 49 = 1 + 2n - 2$
$\Rightarrow 50 = 2n \Rightarrow n = 25$

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get
$S_{25} = \frac{25}{2}(1 + 49) = \frac{25}{2} \times 50 = 25 \times 25 = 625$

Ex 5.3 Question 15.

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Penalty for first day = Rs 200, Penalty for second day = Rs 250
Penalty for third day = Rs 300
It is given that penalty for each succeeding day is Rs 50 more than the preceding day.
It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

So, we have an AP of the form $200, 250, 300, 350 \ldots 30$ terms
First term $= a = 200$, Common difference $= d = 50, n = 30$
Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{30}{2}[400 + (30-1)50]$$
$$\Rightarrow S_n = 15(400 + 29 \times 50)$$
$$\Rightarrow S_n = 15(400 + 1450) = 27750$$

Therefore, penalty for 30 days is Rs. 27750.

Ex 5.3 Question 16.

A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.

Answer.

It is given that sum of seven cash prizes is equal to Rs 700 .
And, each prize is R.s 20 less than its preceding term.
Let value of first prize = Rs. a
Let value of second prize = Rs $(a-20)$
Let value of third prize $= \mathrm{Rs}(a-40)$
So, we have sequence of the form:
$a, a-20, a-40, a-60 \ldots$
It is an arithmetic progression because the difference between consecutive terms is constant.
First term $= a$, Common difference $= d = (a-20) - a = -20$
$n = 7$ (Because there are total of seven prizes)
$S_7 = R_s 700\{$ given $\}$
Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_7 = \frac{7}{2}[2a + (7-1)(-20)]$$
$$\Rightarrow 700 = \frac{7}{2}[2a - 120]$$
$$\Rightarrow 200 = 2a - 120$$
$$\Rightarrow 320 = 2a \Rightarrow a = 160$$

Therefore, value of first prize = Rs 160
Value of second prize $= 160 - 20 =$ Rs 140
Value of third prize $= 140 - 20 =$ Rs 120
Value of fourth prize $= 120 - 20 =$ Rs 100
Value of fifth prize $= 100 - 20 =$ Rs 80
Value of sixth prize $= 80 - 20 =$ Rs 60
Value of seventh prize $= 60 - 20 = \mathrm{Rs}\,40$

Ex 5.3 Question 17.

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Answer.

There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

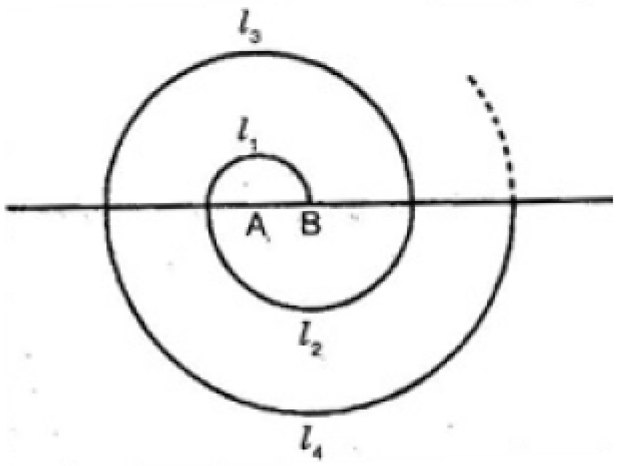
The number of trees planted by class I = number of sections $\times 1 = 3 \times 1 = 3$
The number of trees planted by class II = number of sections $\times 2 = 3 \times 2 = 6$

The number of trees planted by class III = number of sections $\times 3 = 3 \times 3 = 9$
Therefore, we have sequence of the form $3, 6, 9 \ldots 12$ terms
To find total number of trees planted by all the students, we need to find sum of the sequence $3, 6, 9, 12 \ldots 12$ terms.

First term $= a = 3$, Common difference $= d = 6 - 3 = 3$ and $n = 12$
Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get
$S_{12} = \frac{12}{2}[6 + (12-1)3] = 6(6 + 33) = 6 \times 39 = 234$

Ex 5.3 Question 18.

A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii $0.5 \mathrm{~cm}, 1.0 \mathrm{~cm}, 1.5 \mathrm{~cm}, 2.0 \mathrm{~cm}, \ldots$ What is the total length of such a spiral made up of thirteen consecutive semicircles.



Answer.

Length of semi-circle $=\frac{\text{Circumfere nce of circle}}{2}=\frac{2\pi}{2}=\pi$.
Length of semi-circle of radii $0.5 \mathrm{~cm}=\pi(0.5) \mathrm{cm}$
Length of semi-circle of radii $1.0 \mathrm{~cm}=\pi(1.0) \mathrm{cm}$
Length of semi-circle of radii $1.5 \mathrm{~cm}=\pi(1.5) \mathrm{cm}$

Therefore, we have sequence of the form: $\pi(0.5), \pi(1.0), \pi(1.5) \ldots 13$ terms \{There are total of thirteen semi-circles\}.

To find total length of the spiral, we need to find sum of the sequence $\pi(0.5), \pi(1.0), \pi(1.5)$ ... 13 terms

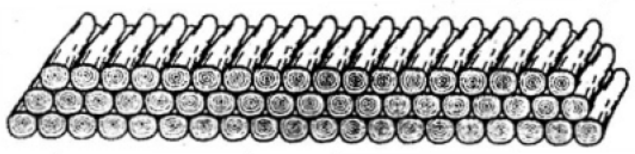
Total length of spiral $=\pi(0.5)+\pi(1.0)+\pi(1.5) \ldots 13$ terms
$\Rightarrow$ Total length of spiral $=\pi(0.5+1.0+1.5) \ldots 13$ terms ... (1)
Sequence $0.5, 1.0, 1.5 \ldots 13$ terms is an arithmetic progression.
Let us find the sum of this sequence.
First term $=\mathrm{a}=0.5$, Common difference $=1.0-0.5=0.5$ and $\mathrm{n}=13$

Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get
$S_{13}=\frac{13}{2}[1+(13-1)0.5]=6.5(1+6)=6.5\times 7=45.5$

Therefore, $0.5+1.0+1.5+2.0 \ldots 13$ terms $=45.5$
Putting this in equation (1), we get
Total length of spiral $=\pi(0.5+1.5+2.0+\ldots 13$ terms $)=\pi(45.5)=143 \mathrm{~cm}$

Ex 5.3 Question 19.

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Answer.

The number of logs in the bottom row $=20$
The number of logs in the next row $=19$
The number of logs in the next to next row $=18$
Therefore, we have sequence of the form $20, 19, 18 \ldots$
First term $=a=20$, Common difference $=d=19-20=-1$
We need to find that how many rows make total of 200 logs.
Applying formula, $S_n=\frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get
$200=\frac{n}{2}[40+(n-1)(-1)]$
$\Rightarrow 400=n(40-n+1)$
$\Rightarrow 400=40n-n^2+n$
$\Rightarrow n^2-41n+400=0$

It is a quadratic equation, we can factorize to solve the equation.
$\Rightarrow n^2-25n-16n+400=0$
$\Rightarrow n(n-25)-16(n-25)=0$
$\Rightarrow (n-25)(n-16)$
$\Rightarrow n=25,16$

We discard $n=25$ because we cannot have more than 20 rows in the sequence. The sequence is of the form: $20, 19, 18 \ldots$

At most, we can have 20 or less number of rows.

Therefore, $\mathrm{n}=16$ which means 16 rows make total number of logs equal to 200 .

We also need to find number of logs in the 16 th row.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

$200 = 8(20 + l)$

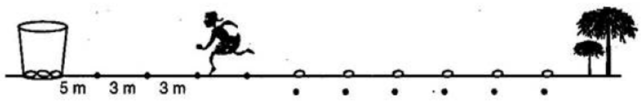
$\Rightarrow 200 = 160 + 8I$

$\Rightarrow 40 = 8l \Rightarrow l = 5$

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

Ex 5.3 Question 20.

In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Answer.

The distance of first potato from the starting point $= 5$ meters

Therefore, the distance covered by competitor to pick up first potato and put it in bucket $= 5 \times 2 = 10$ meters

The distance of Second potato from the starting point $= 5 + 3 = 8$ meters \{All the potatoes are 3 meters apart from each other\}

Therefore, the distance covered by competitor to pick up 2nd potato and put it in bucket $= 8 \times 2 = 16$ meters

The distance of third potato from the starting point $= 8 + 3 = 11$ meters

Therefore, the distance covered by competitor to pick up 3rd potato and put it in bucket = $11 \times 2 = 22$ meters

Therefore, we have a sequence of the form $10, 16, 22 \ldots 10$ terms

(There are ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find:

$10 + 16 + 22 + \ldots 10 \text{ terms}$

First term $= a = 10$, Common difference $= d = 16 - 10 = 6$

$n = 10\{$ There are total of 10 terms in the sequence $\}$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP, we get

$S_{n10} = \frac{10}{2}[20 + (10 - 1)6] = 5(20 + 54) = 5 \times 74 = 370$

Therefore, total distance covered by competitor is equal to 370 meters.

NCERT Solutions Class 10 Maths Chapter 5: Arithmetic Equations

Ex 5.4 Question 1.

Which term of the AP: $121, 117, 113, \ldots..$ is its first negative term?

Answer.

Given: $121, 117, 113, \ldots\ldots$
Here $a = 121, d = 117 - 121 = -4$

$$\begin{aligned} &\text{Now, } a_n = a + (n-1)d \\ &= 121 + (n-1)(-4) \\ &= 121 - 4n + 4 = 125 - 4n \end{aligned}$$

For the first negative term, $a_n < 0$

$$\begin{aligned} &\Rightarrow 125 - 4n < 0 \\ &\Rightarrow 125 < 4n \\ &\Rightarrow \frac{125}{4} \qquad \Rightarrow 31\frac{1}{4}n \text{ is an integer and } n > 31\frac{1}{4} \end{aligned}$$

n is an integer and $n > 31\frac{1}{4}$.
Hence, the first negative term is 32^{nd} term.

Ex 5.4 Question 2.

The sum of the third and the seventh terms of an AP is 6 and their product is 8 . Find the sum of sixteen terms of the AP.

Answer.

Let the AP be $a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, \ldots.$
Then, $a_3 = a - 2d, a_7 = a + 2d$

$$\begin{aligned} &\Rightarrow a_3 + a_7 = a - 2d + a + 2d = 6 \\ &\Rightarrow 2a = 6 \\ &\Rightarrow a = 3 \ldots\ldots\ldots. \text{ (i)} \end{aligned}$$

Also $(a-2d)(a+2d) = 8$

$$\begin{aligned} &\Rightarrow a^2 - 4d^2 = 8 \\ &\Rightarrow 4d^2 = a^2 - 8 \\ &\Rightarrow 4d^2 = 3^2 - 8 \\ &\Rightarrow 4d^2 = 1 \\ &\Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2} \end{aligned}$$

Taking $d=\frac{1}{2}$,

$$S_{16}=\frac{16}{2}[2\times(a-4d)+(16-1)d]$$
$$=8\left[2\times\left(3-4\times\frac{1}{2}\right)+15\times\frac{1}{2}\right]$$
$$=8\left[2+\frac{15}{2}\right]=8\times\frac{19}{2}=76$$

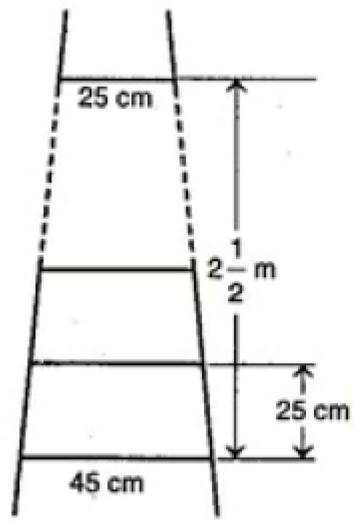
Taking $d=\frac{-1}{2}$,

$$S_{16}=\frac{16}{2}[2\times(a-4d)+(16-1)d]$$
$$=8\left[2\times\left(3-4\times\frac{-1}{2}\right)+15\times\frac{-1}{2}\right]$$
$$=8\left[\frac{20-15}{2}\right]=8\times\frac{5}{2}=20$$

$\therefore S_{16}=20$ and 76

Ex 5.4 Question 3.

A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}\mathrm{m}$ apart, what is the length of the wood required for the rungs?



Answer.

Number of rungs $(n)=\frac{2\frac{1}{2}\times 100}{25}=10$

The length of the wood required for rungs = sum of 10 rungs

$=\frac{10}{2}[25+45]=5\times 70=350\ \mathrm{cm}$

Ex 5.4 Question 4.

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Answer.

Here $a=1$ and $d=1$

$$\therefore S_{x-1}=\frac{x-1}{2}[2\times 1+(x-1-1)\times 1]$$
$$=\frac{x-1}{2}(2+x-2)$$
$$\frac{(x-1)x}{2}=\frac{x^2-x}{2}$$
$$S_x=\frac{x}{2}[2\times 1+(x-1)\times 1]$$
$$=\frac{x}{2}(x+1)=\frac{x^2+x}{2}$$
$$S_{49}=\frac{49}{2}[2\times 1+(49-1)\times 1]$$
$$=\frac{49}{2}(2+48)=49\times 25$$

According to question,

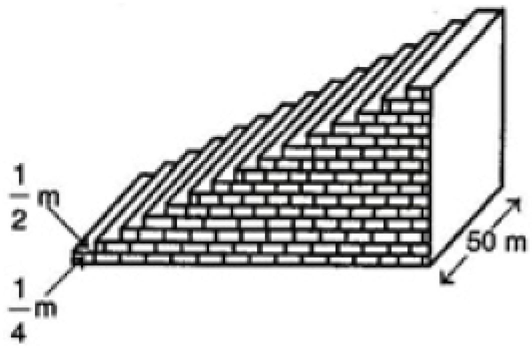
$$S_{x-1}=S_{49}-S_x$$
$$\Rightarrow\frac{x^2-x}{2}=49\times 25-\frac{x^2+x}{2}$$
$$\Rightarrow\frac{x^2-x}{2}+\frac{x^2+x}{2}=49\times 25$$
$$\Rightarrow\frac{x^2-x+x^2+x}{2}=49\times 25$$
$$\Rightarrow x^2=49\times 25$$
$$\Rightarrow x^2=49\times 25$$
$$\Rightarrow x=\pm 35$$

Since, x is a counting number, so negative value will be neglected.

$\therefore x=35$

Ex 5.4 Question 5.

A small terrace at a football ground comprises of 15 steps each of which is $50 \mathrm{~m}$ long and built of solid concrete.



Each step has a rise of $\frac{1}{4} \mathrm{~m}$ and a tread of $\frac{1}{2} \mathrm{~m}$ (see figure). Calculate the total volume of concrete required to build the terrace.

Answer.

Volume of concrete required to build the first step, second step, third step, ... (in m^2) are

$\frac{1}{4} \times \frac{1}{2} \times 50,\left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50,\left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \ldots .$

$\Rightarrow \frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \ldots \ldots .$

$\therefore$ Total volume of concrete required $=\frac{50}{8}+2 \times \frac{50}{8}+3 \times \frac{50}{8}+\ldots \ldots .$

$=\frac{50}{8}[1+2+3+\ldots \ldots .]$

$=\frac{50}{8} \times \frac{15}{2}[2 \times 1+(15-1) \times 1][\because n=15]$

$=\frac{50}{8} \times \frac{15}{2} \times 16$

$=750 \mathrm{~m}^3$